Towards Quantum Plasma Simulation: Benchmarking Variational Quantum Algorithms for Electromagnetic Wave Propagation

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1. Introduction & Motivation

Quantum computing offers a promising approach to solving eigenvalue problems that arise in electromagnetic waveguide analysis. **Variational Quantum Algorithms (VQAs)** are hybrid quantum-classical methods that distribute computational tasks between both architectures.

Key idea: Any problem formulated as a Hamiltonian system can be solved via the **Variational Quantum Eigensolver (VQE)** algorithm:

$$F_0(\boldsymbol{\theta}) = \langle \psi(\boldsymbol{\theta})|\, M \,|\psi(\boldsymbol{\theta})\rangle \geq E_0$$

This work:

1. Reproduce VQE for rectangular waveguide **Tranverse Electric/Magnetices (TE/TM)** modes
2. **Extend** to **cold plasma-filled** waveguides (O-mode)
3. Develop **ML warm-start** to accelerate convergence

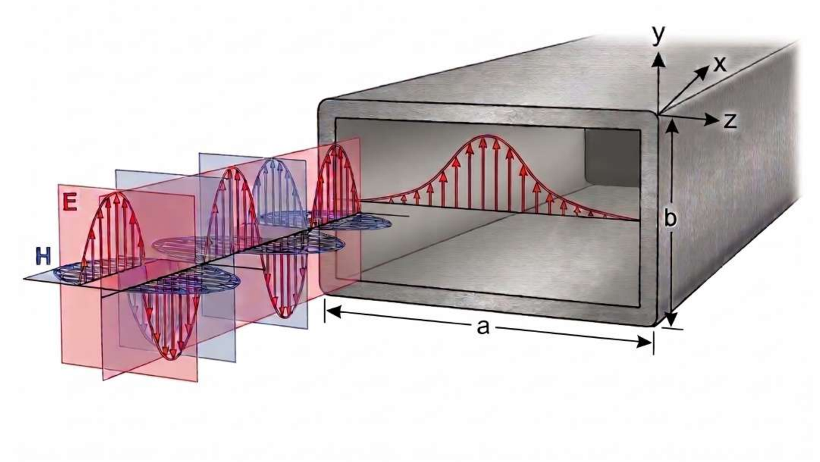


Figure: Physical representation of our system.

2. Methodology: VQE for Waveguides

Helmholtz equation via finite differences for a rectangular metallic waveguide:

$$\nabla_s^2 H_z = -k_s^2 H_z \quad \text{(TE modes)}$$
$$\nabla_s^2 E_z = -k_s^2 E_z \quad \text{(TM modes)}$$

Discretisation yields an eigenvalue problem: $\mathbf{Mv} = \lambda \mathbf{v}$, with M in the TM case:

$$M = \frac{1}{\Delta x^2}\begin{pmatrix} 3 & -1 & 0 & \cdots & & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & & \\ \vdots & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & \cdots & 0 & 0 & -1 & 3 \end{pmatrix}$$

and with 1 in the first and last element of the diagonal for the TE case. **Pauli decomposition** of M enables quantum evaluation trough the Harware Efficient Ansatz:

$$M_j = 4\, I^{\otimes(n_x+n_y)} + \sum_{i=1}^{2} H_i + \sum_{i=3}^{5} V^\dagger H_i V + \sum_{i=6}^{8} W^\dagger H_i W$$

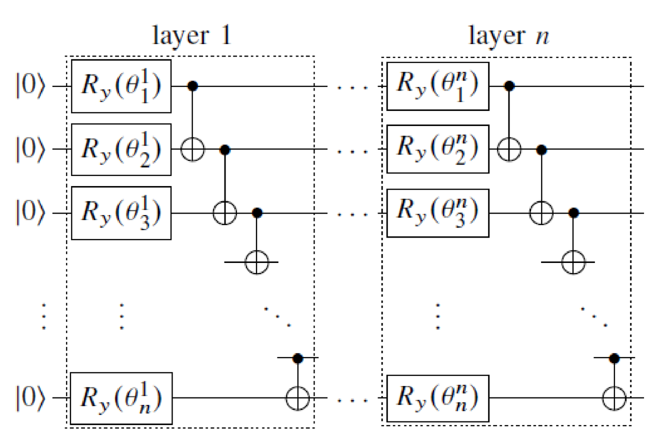


Figure: Hardware Efficient Ansatz Diagram

Higher modes via penalty method:

$$F_k(\boldsymbol{\theta}) = \langle \psi(\boldsymbol{\theta})|M|\psi(\boldsymbol{\theta})\rangle + \sum_{i=0}^{k-1} \beta_i \left|\langle \psi(\boldsymbol{\theta})|\psi^{(i)}\rangle\right|^2$$

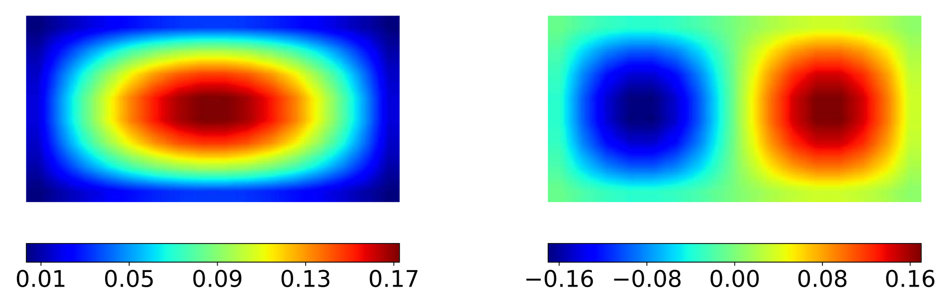
3. Results: Vacuum Waveguide

Waveguide: 15 mm × 10 mm, various (n_x, n_y) configurations, with the main focus on a NISQ configuration such as $(n_x, n_y) = 4, 3$.

We compare the final frequency we obtain with respects to the classical eigensolvers and also the theoretical value. We can see that it converges up to the 4th decimal with the classical eigensolvers and with an error of 1% with the theoretical ones.

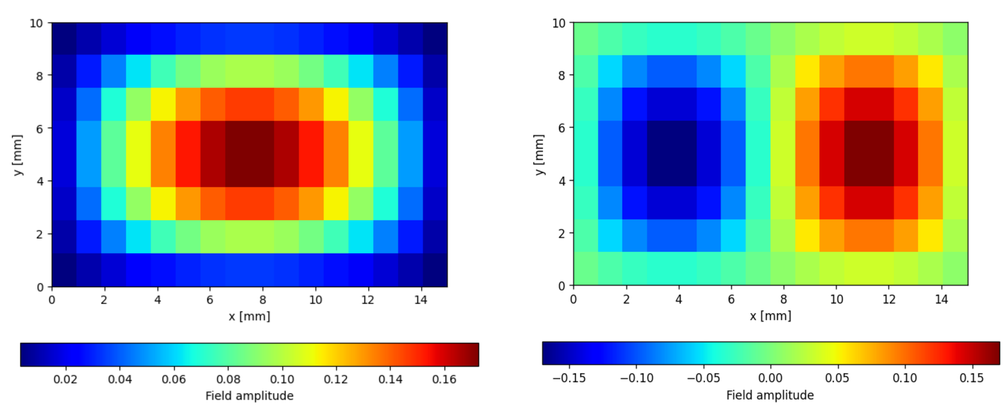
Mode	VQA (GHz)	Classical (GHz)	Theoretical (GHz)
TE_{10}	9.9770	9.9770	9.9931
TE_{01}	14.8935	14.8935	14.9896
TM_{11}	17.9264	17.9264	18.0153
TM_{21}	24.8225	24.8225	24.9827

Table 1: VQA matches classical solver precision; both converge to analytical values as grid refines.



(a) Ground State field. (b) Excited state field.

Figure: Theoretical results from paper [1].



(a) Ground State field. (b) Excited state field.

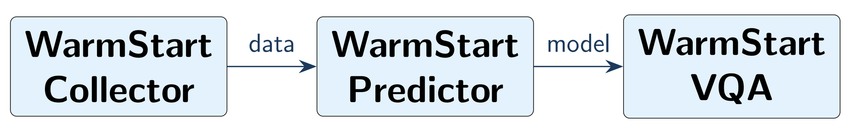
Figure: Results obtained from our VQA algorithm.

4. Machine learning warm start model

Due to the random nature of the VQE we have 3 types of solutions: correct solutions, solutions that converge to a higher mode, incorrect solutions.

- **Sweet spot**: convergence > 80% when layers ≈ total qubits.
- Higher modes require **more layers** to avoid local minima and thus making simulating higher modes more complex and computationally demanding.

Solution: Train an MLP neural network to predict good initial θ_0 from problem features.



Feature vector (6D):

Index	Feature
0	n_x (x-direction nº of qubits)
1	n_y (y-direction nº of qubits)
2	n_{layers} (ansatz depth)
3	Mode type: 0 = TM, 1 = TE
4	Mode state index k
5	$\log_{10}(1 + N_e)$ (density of plama profile)

Training data is **filtered**: only VQE runs with $\frac{|\lambda_{VQE} - \lambda_{ref}|}{\lambda_{ref}} <$ 5% are kept. The model falls back to random initialisation if no trained predictor is available. We obtain an average speedup of 20%. It is not a high speedup, but many vectors have redundant data and varied starting points preventing the model from identifying meaningful patterns.

5. Extension: Cold Plasma Waveguide

As has been suggested [2], we extend our VQE to plasma formalism. For a **single-species cold plasma** with $\vec{B} = B_0\hat{y}$, the ordinary (O-mode) wave satisfies a modified Helmholtz equation:

$$\left(-\partial_{xx} - \partial_{yy} + \frac{\omega_p^2(x,y)}{c^2}\right) E_z = \frac{\omega^2}{c^2} E_z$$

where $\omega_p^2(x,y) = \frac{q_e^2 N_e(x,y)}{m_e \varepsilon_0}$ with a **Gaussian plasma density** profile. This formulation is proposed in [3].

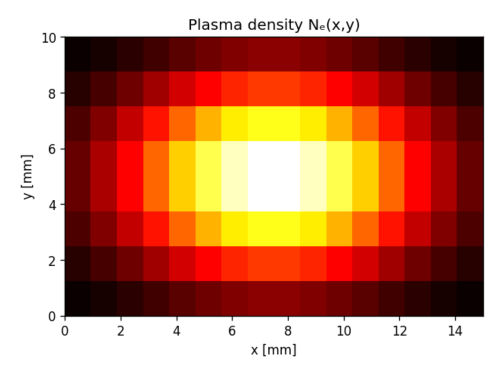
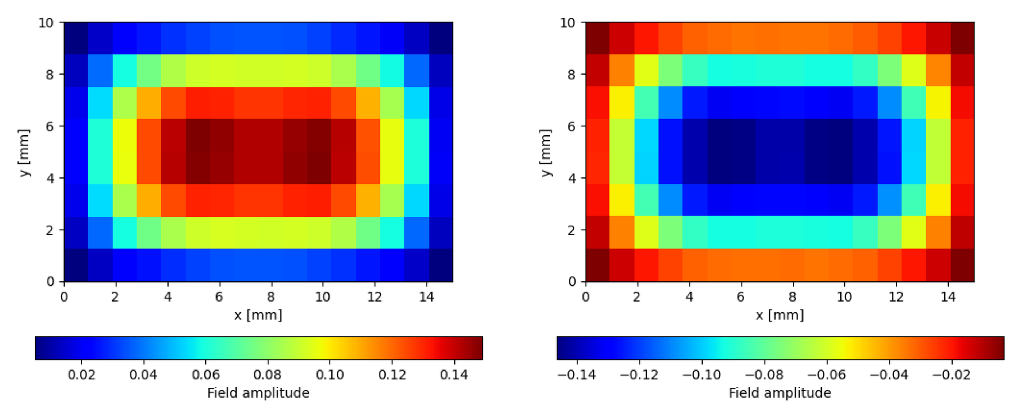


Figure: Plasma density Gaussian profile distribution.

The plasma term adds a **diagonal matrix** to the Hamiltonian, preserving the VQE framework.

We can look now at some physical insight to check the behaviour of our model. In theory the mode profile is modified by the central plasma density — field is pushed toward the waveguide walls when ω_p increases. We model 2 cases, 1 in which the plasma frequency is close but bigger than the wave frequency and another in which the plasma frequency is similar but smaller.



(a) Transmission with $\omega > \omega_p$. (b) Reflecting wave $\omega < \omega_p$.

Figure: Results in the cold plasma case with 2 cases of different plasma densities.

We obtain the expected results, when $\omega < \omega_p$, the refraction index is complex meaning that the wave is evanescence and absorption will happen. And when $\omega > \omega_p$ there is wave propagation across the plasma waveguide.

7. Conclusions & Outlook

- ✓ VQE **reproduces classical eigenvalue results** for rectangular waveguide TE/TM modes with high fidelity
- ✓ Successfully **extended to cold plasma** (O-mode): captures propagation vs. evanescence transition
- ✓ **ML warm-start** reduces the need for random restarts
- → **Future:** X-mode (extraordinary wave), larger qubit counts, real quantum hardware

References

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